Funding Value Adjustment and Valuing Interest Rate Swaps
Msc Thesis Financial Econometrics

Author: Hidde Schwietert
Supervisor: Cees Diks

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Abstract
This thesis deals with the incorporation of both credit and funding risk in derivative valuation. The focus is on interest rate swaps for which an innovative framework is provided for modelling funding value adjustment (FVA) along with bilateral credit value adjustment (BCVA). In doing so, a case study is examined where a bank and a corporate enter into a plain vanilla interest rate swap. Underlying risk factors such as interest rates and credit spreads are modelled by stochastic processes while FVA is evaluated according to a Monte Carlo integration. In addition, we analyse the impact of wrong-way risk, credit spread levels and credit spread volatilities on FVA. FVA is found to have significant impact on derivatives valuation. Besides, FVA is found to be quite sensitive to changes in credit spread levels and volatilities. Hence, it is important to model thoroughly such dynamic features.

Keywords: Counterparty Credit Risk, Funding Risk, Bilateral Credit Value Adjustment, Funding Value Adjustment, Interest Rate Swaps.
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Chapter 1

Introduction

The global credit crisis resulted in regulatory and accounting changes regarding derivative valuation. In January 2013, International Financial Reporting Standards 13 became effective, forcing banks and other derivative dealers to incorporate credit value adjustment (CVA) and debt value adjustment (DVA) in their fair derivative valuations. While the incorporation of CVA and DVA has been widely studied and accepted by both practitioners and academics, the focus is now on the calculation and relevance of funding value adjustment (FVA). The occurrence of FVA is highly linked with the financial crisis. Prior to the crisis, LIBOR was often regarded as a suitable proxy for the risk-free rate. Hence, collateral rates were commonly LIBOR based (Hull & White, 2013) and so funding costs were offset. Through this, there was no need for a FVA. However, post crisis banks’ are not considered as default free any more, hence the overnight indexed swap rate (OIS rate) has become benchmark for risk-free rates. As a result, collateral rates have become mostly OIS based. Thus, as funding has become relatively more costly and the interest received on collateral no longer offsets the funding cost, the FVA has become non-negligible. Reflecting funding cost into the valuation of derivatives has become a paramount topic in the financial industry. And yet, the literature is still limited. At the moment there is no standard procedure involving both counterparty credit risk, own default risk and funding cost into the valuation of a derivative contract.

Hull & White (2012), show in a Black-Scholes framework that implementing FVA conflicts with fundamental derivative pricing theory and believe FVA should not be taken into account when pricing derivatives. However, many practitioners criticize the approach taken by Hull & White (2012), claiming funding costs are real and can dramatically impact banks profit and loss statements. Furthermore, they argue that fundamental assumptions made in the Black-Scholes model do not hold in reality, hence funding cost should be taken into account. Recently some attempts have been made to implement CVA, DVA and FVA into the valuation of derivatives. Morini & Prampolini (2010), show in a simple modeling setting involving a lender and a borrower, that the crucial variable determining the lender’s net funding cost is the bond-CDS basis, leaving extensions to general derivative payouts for future research. Other more comprehensive attempts have been made by Brigo et al. (2014) and Crépey et al. (2013), both showing that the incorporation of FVA leads to a non-linear recursive pricing problem. In this thesis we provide an innovative framework for modeling funding value adjustment, extending the current literature by modeling the relevant funding spread separately with a
stochastic model. Besides, the focus in this thesis will be on interest rate swaps. Interest rate swaps are popular, highly liquid derivatives and therefore interesting cases to examine the implementation of FVA. The purpose of this thesis is to investigate the incorporation of credit risk and funding cost in the valuation of (plain vanilla) interest rate swaps. Hence, the research question investigated in this thesis is how can FVA be included in the fair valuation of plain vanilla interest rate swaps along with CVA and DVA? Furthermore, the impact of wrong-way risk, credit spread levels and credit spread volatilities will be examined and discussed in the form of an impact analysis.

In order to analyze FVA in the fair valuation of interest rate swaps, we examine a case study where Deutsche Bank AG and Koninklijke Ahold NV enter into a plain vanilla interest rate swap, exchanging a fixed rate for the Euribor rate. Moreover, we want to adjust the price of this swap for counterparty credit risk and funding risk by calculating CVA, DVA and FVA. In order to calculate these value adjustments we need to model all underlying risk factors regarding the interest rate swap. To start with, we model the Euribor rate according to G2++ model. This enables us to calculate exposure profiles regarding the swap used to evaluate the value adjustments. Thereby, we use Cox-Ingersoll-Ross (CIR) dynamics to model the Euribor-OIS spread. In this way we model the OIS rate, i.e. the Eonia rate, which is used for discounting. Besides, default intensities of both Deutsche Bank and Ahold are modelled according to a CIR++ model. Lastly, the net funding costs of Deutsche Bank are also modelled according to a CIR process and used to calculate FVA. We use real market data to calibrate our models. The data are obtained from Bloomberg, observed on April 30, 2014. Furthermore, Euler discretization and Hypersphere decomposition are used for the numerical simulation of our stochastic processes. The CVA, DVA and FVA are finally calculated by Monte Carlo integration. Thereby, as the calculations of BCVA and FVA are highly model dependent, we perform an impact analysis. In particular, we analyse the impact of wrong-way risk, credit spread levels and credit spread volatilities on BCVA and FVA.

The thesis is structured as follows. In Chapter 2 basic relevant theory is discussed, including short rates, discount factors, forward rates and interest rate swaps. Thereby, we describe the occurrence and usage of credit, debt and bilateral credit value adjustment, while the chapter ends with an extensive review of the FVA debate. Chapter 3 gives an overview of the methodology used in this thesis. This includes model specifications with the corresponding calibration and simulation methods. Besides, the framework for modelling FVA is discussed in Section 3.5. In Chapter 4 we illustrate a case study, specifying the conditions regarding the swap, the relevant market data and the research approach. The results of both the case study and the impact analysis are presented in Chapter 5. Finally, Chapter 6 concludes.
Chapter 2

Theory

In this chapter, basic relevant theory is discussed, with the aim to help the reader to understand the remaining of this thesis. We begin by describing basic interest rate theory including short rates, discount factors, interest rate swaps and forward rates. Thereafter, we describe the occurrence and usage of credit, debt and bilateral credit value adjustment. Lastly, we focus on FVA and provide an extensive overview of the FVA debate.

2.1 Fundamentals

In this section, basic building blocks regarding financial instruments are defined. To start with, define a risk-free bank account \( B(t) \) at time \( t \geq 0 \), representing a risk-less investment where profit is accrued at a stochastic continuously compounded interest rate \( r_t \). We will call \( r_t \) the instantaneous rate or short rate. Thereby, we assume that \( B(0) = 1 \) and that the dynamics of the bank account correspond to

\[
\frac{dB(t)}{B(t)} = r_t dt. \tag{2.1}
\]

As a consequence, the value of a unit amount invested in the bank account at time \( t = 0 \) equals

\[
B(t) = e^{\int_0^t r_u du}, \tag{2.2}
\]

at time \( t \). For discounting a unit amount at \( T \) back to \( t \) we use the stochastic discount factor \( D(t,T) \), which is given by

\[
D(t,T) = \frac{B(t)}{B(T)} = e^{\int_t^T r_u du}. \tag{2.3}
\]

The discount factor can be seen as a random variable at time \( t \), hence it is referred to as stochastic. Furthermore, we denote by \( \tau(t,T) \) the year fraction, which is the chosen time measure between \( t \) and \( T \). Which can in general vary for different day-count conventions, i.e. the particular choice to measure the time between two dates. In our calculations, we use \( \tau(t,T) = T - t \).

A zero coupon bond with maturity date \( T \), is a contract that guarantees its holder the payment of a unit amount of currency at time \( T \), in absence of any intermediate (coupon) payments. The value of
the zero coupon bond at time $t < T$ is denoted by $P(t, T)$. Furthermore, we denote by $L(t, T)$ the *simply-compounded spot interest rate*, prevailing at time $t$ for the maturity $T$. Starting from $P(t, T)$ units of currency at time $t$, the simply-compounded spot interest rate is the constant rate at which a unit amount of currency is produced at maturity. In formulas:

$$P(t, T)(1 + L(t, T)\tau(t, T)) = 1,$$

hence a zero coupon bond price can be expressed as

$$P(t, T) = \frac{1}{1 + L(t, T)\tau(t, T)}.$$

The market *LIBOR rate* (London Interbank Offered Rate) is a simply-compounded interest rate at which a selection of leading banks in London can borrow in interbank transactions. Pre-crisis, these LIBOR panel banks were commonly assumed to be default-free. Hence, LIBOR rates were an important benchmark for risk-free rates.

### 2.2 Interest Rate Swaps

The derivative examined in this thesis is the interest-rate swap (IRS). Interest rate swaps are commonly used for both hedging and speculating. In this section we explain basic concepts regarding interest-rate swaps by introducing forward rates, giving the definition of IRSs, deriving the pricing formula and introduce swap rates.

#### 2.2.1 Forward Rates

Forward rates can be defined through *forward-rate agreements* (FRAs). Basically, a FRA is a contract to fix a future uncertain (floating) rate and is characterized by three time instants: the valuation time $t$, time $T > t$ and tenor $S > T$. The holder of the contract is allowed to exchange a floating spot rate $L(T, S)$ for a fixed interest rate $K$ between $T$ and $S$. Consequently, per unit amount of currency at time $t$ one receives $K\tau(T, S)$ units of currency and pays $\tau(T, S)L(T, S)$ at time $S$. Assuming both rates have the same day-count convention, the total pay-off at time $S$ equals:

$$\tau(T, S)(K - L(T, S)).$$

The FRA is a fair contract at time $t$ when its present value equals zero. The value of $K$ such that the present value of the FRA contract equals zero is called the *simply-compounded forward interest rate*, which we denote by $F$. In Section 2.1.3 of [Brigo & Mercurio (2006)](https://doi.org/10.1007/978-3-540-34769-8) it is shown that the simply-compounded forward interest rate is

$$F(t; T, S) = \frac{1}{\tau(T, S)} \left( \frac{P(t, T)}{P(t, S)} - 1 \right).$$
2.2.2 Interest Rate Swap Valuation and Swap Rates

A more extensive form of a FRA (considered in Section 2.2.1) is the IRS. Basically, the IRS is an agreement in which two parties exchange interest rate cash flows over an agreed period of time. Parties may exchange different floating rates, as well as floating for fixed rates (or vice versa). Consider a *plain vanilla interest rate swap*, which is an agreement to exchange fixed interest rate payments for floating interest rate payments. The fixed rate is specified in the contract, whereas the floating rate is a varying interest rate such as the LIBOR rate. The *fixed leg* is the collection of fixed payments, whereas the *floating leg* is the collection of floating payments. A *payer swap* is an IRS in which the fixed leg is paid and the floating leg is received. Contrariwise, the holder of a *receiver swap* pays the floating leg and receives the fixed leg.

To ease the notation, we consider fixed-rate payments and floating-rate payments occur at the same dates and with the same year fraction (generalizations to different payment dates and day-count conventions are trivial). Assume a payer IRS pre specifies a set of payment dates \( T \) exchanging fixed for floating payments at times \( T = \{ T_a, ..., T_b \} \). Denote by \( \tau_i \) the year fraction between \( T_{i-1} \) and \( T_i \) and set \( \tau = \{ \tau_{a+1}, ..., \tau_b \} \). We take a unit national on the swap. In this case, at every time instant \( T_i \) the fixed leg pays out \( -\tau_i K \), whereas the floating leg pays \( \tau_i L(T_{i-1}, T_i) \). The discounted pay-off for a payer IRS at a time \( t < T_a \) is

\[
\sum_{i=a+1}^{b} D(t, T_i) \tau_i (L(T_{i-1}, T_i) - K).
\]

(2.8)

Trivially, the discounted pay-off for a receiver IRS at time \( t < T_a \) equals

\[
\sum_{i=a+1}^{b} D(t, T_i) \tau_i (K - L(T_{i-1}, T_i)).
\]

(2.9)

One may recognize this last pay-off as a portfolio of FRAs. Hence, the value of a receiver IRS (RFS) can be written as

\[
RFS(t, T, \tau, K) = \sum_{i=a+1}^{b} FRA(t, T_{i-1}, T_i, \tau_i, K)
\]

\[
= \sum_{i=a+1}^{b} \tau_i D(t, T_i) (K - F(t; T_{i-1}, T_i))
\]

(2.10)

Where the symbol \( RFS \) originates from the fact that a receiver swap is more formally a receiver forward-starting swap. In the same way, a payer swap is denoted by \( PFS \). Furthermore, we call the particular value of \( K \) for which \( RFS(t, T, \tau, K) = 0 \) the *forward swap rate* \( S_{a,b}(t) \) and obtain

\[
S_{a,b}(t) = \frac{\sum_{i=a+1}^{b} \tau_i D(t, T_i) F(t; T_{i-1}, T_i)}{\sum_{i=a+1}^{b} \tau_i D(t, T_i)}.
\]

(2.11)

Note that in the above derivation no Libor rate derived products are used to represent the term structure of risk-free rates. This relates to the post-crisis view, in which Libor banks are not assumed
to be default-free anymore. Indeed, the OIS rate has become the benchmark for risk-free rates (Hull & White, 2013). Therefore, the forward rates and discount rates in Eq. 2.11 should be bootstrapped from multiple distinct yield curves as in Bianchetti (2010).

2.3 Counterparty Credit Risk

In Basel II counterparty credit risk is defined as: the risk that the counterparty to a transaction could default before the final settlement of the transactions cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default. Therefore, over-the-counter (OTC) derivatives are subject to counterparty credit risk, since there is no party that guarantees the cash flows agreed on the contract. Due to the global credit crisis, scenarios in which large counterparties may default have become considered to be more realistic. Hence, the necessity to reflect counterparty credit risk in the valuation of derivatives materialized.

2.3.1 Unilateral Credit Value Adjustment

Unilateral credit value adjustment considers transactions seen from the point of view of a pricing counterparty, facing a default risky counterparty. The pricing default-free counterparty (A) incorporates the credit risk of the default risky counterparty (B), by introducing a credit value adjustment (CVA). This adjustment should be subtracted from the value of the derivative contract assuming no default risk for both counterparties. Let us denote by $\bar{V}_A$ the adjusted value of the derivative, subject to counterparty default risk. The subscript $A$ denotes that the value is derived from the point of view of $A$. We denote by $V_A$ the analogous quantity when counterparty risk is not considered. Then, $\bar{V}_A = V_A - CVA_A$. On the contrary, if counterparty B would price the derivative $\bar{V}_B = V_B - CVA_B$, where we assume $V_A = V_B$. Commonly, the two counterparties have different credibility $CVA_A \neq CVA_B$, hence $\bar{V}_A \neq \bar{V}_B$ (Brigo et al., 2011). In this case, the counterparty risk valuation problem is said to be asymmetric for two parties $A$ and $B$ (Brigo et al., 2013). Hence, the two parties do not agree on the price of a derivative contract including credit risk.

2.3.2 Bilateral Credit Value Adjustment

A trend that has become more relevant and popular, particularly since the global credit crisis, has been to integrate the bilateral nature of counterparty credit risk. This involves that a pricing counterparty would consider a CVA calculated under the assumption that they, as well as their counterparty, may default. Hence, to make the counterparty risk valuation problem in Section 2.3.1 symmetric both counterparties have to incorporate a debt value adjustment (DVA) term to adjust for their own risk of default. Understandably, the DVA term calculated by $A$ has to equal the CVA term calculated by $B$ and vice versa. So, $DVA_A = CVA_B$ and $DVA_B = CVA_A$, has to hold consistently. Incorporating this DVA term leads to a bilateral credit value adjustment (BCVA). Both counterparties will mark a positive CVA to be subtracted and a positive DVA to be added to the
value of the derivative assuming no default risk for both counterparties. Hence, \( \hat{V}_A = V_A + BCVA_A \), where \( BCVA_A = DVA_A - CV_A \). Since by consistency \( BCVA_B = DVA_B - CV_B \), we can write the following chain of equalities:

\[
\begin{align*}
\hat{V}_B &= V_B + BCVA_B \\
&= -V_A - CV_A + DVA_B \\
&= -V_A + CV_A - DVA_A \\
&= -(V_A + BCVA_A) \\
&= -\hat{V}_A.
\end{align*}
\]

Hence, price symmetry is obtained and the two parties agree on the price of a derivative contract including credit risk.

### 2.4 the FVA Debate

#### 2.4.1 Funding Costs

Funding costs are interest rate payments, paid by financial institutions for the financial resources (funds) that they deploy in their business. For example, consider a trader who needs to manage a trading position. The trader needs cash to hedge his position, post collateral etcetera. To acquire cash for these operations from either the money market or an internal treasury department, interest payments have to be made. As borrowing cash has a cost, we refer to these interest payments as funding costs. On the other hand, the trader may receive cash in the form of close-out payments, coupons or collateral received. These benefits will generate interest, since a trader would not lend capital for free. In this case, the trader has a funding benefit. Funding value adjustment (FVA) is, in line with CVA and DVA, an adjustment to the value of a derivative contract to incorporate funding costs or benefits. In our example, FVA reflects the excess or shortfall of cash arising from the traders’ derivatives operations. In formulas: \( FVA = FBA - FCA \), where \( FCA \) and \( FBA \) denote funding cost and funding benefit adjustment respectively.

The occurrence of FVA is highly linked with the global financial crisis. As already mentioned, prior to the crisis, LIBOR (or Euribor) was often regarded as the risk-free interest rate and banks were generally regarded as ‘to big to fail’. Therefore, interest received on posted collateral was commonly LIBOR and so funding costs were offset. Nowadays, banks are not considered as default-free any more. As a consequence, the overnight indexed swap rate (OIS rate), which is typically the rate for overnight unsecured lending between banks (for example the Federal funds rate for US dollars and Eonia for euros), has become the benchmark rate for crediting interest on posted collateral (Hull & White, 2013). Indeed, as illustrated in Figure 2.1, pre-crisis Euribor-OIS spread levels were very small compared to post-crisis Euribor-OIS spread levels. So, as funding has become relatively more costly and the interest received on collateral no longer offsets the funding cost, the FVA has become non-negligible. This explains why FVA has been taking an increasingly important role in valuing
derivatives since the crisis.

Figure 2.1: Graph from Bloomberg where the white line and orange line indicate respectively the 3 month Euribor and OIS spot rate over the past ten years. Obviously, dissimilarities have appeared between the Euribor and OIS rate since the global credit crisis.

At the moment there is no standard procedure to include counterparty credit risk and funding cost in the fair valuation of a contract and the matter has not only been the centre of heated debates between quantitative analysts, traders treasures, risk managers and academics.

2.4.2 The Black and Scholes Argument

Black & Scholes (1973) showed that if one is able to lend and borrow at the risk-free interest rate, the pay-off of an option can be replicated by a portfolio of stock and risk-free debt. This allowed them to derive a partial differential equation, which estimates the price of an option over time. The same partial differential equation, can also be derived by discounting the expected pay-off of the option in a risk-neutral world at the risk-free rate of interest (Merton, 1998). Hull & White (2012) use this fundamental derivative pricing theory to argue that FVA should not be incorporated in the valuation of derivatives. Firstly, they want to avoid confusion about why the risk-free rate is used for discounting when valuing derivatives. They state that we do not discount at a risk-free rate because a bank can fund the derivative at the risk-free rate. We discount at the risk-free rate because this is required by the risk-neutral valuation principle, which should give the correct economic value of a derivative taking into account all its market risks. Secondly, they claim that the funding of hedges is a invalid reason for an FVA: ‘hedging instruments involve buying or selling assets for their market prices and are, therefore, zero net present value investments. As a result, the decision to hedge does not affect valuations’. Thirdly, Hull & White (2012) reason pricing should be kept separate from funding. Therefore, discount rates used to value a project, should depend on the risk of the project rather than the riskiness of the firm that undertakes it. Fourthly, Hull & White (2012) split up DVA in two parts: DVA$_1$ and DVA$_2$. DVA$_1$ refers to the DVA arising from a possible default on its derivatives portfolio and DVA$_2$ is the FVA arising because a dealer may default on its other liabilities (e.g. short term and long term debt). They define $\Delta$DVA as the increase in DVA$_2$ resulting from
the funding requirement of a derivatives portfolio with a particular counterparty. Assuming that the CDS-bond basis is zero implies \( FVA = \Delta DVA \). Lastly, Hull & White (2012) argue \( DVA \) does not need to be calculated for most liabilities at inception because it is already reflected in the market prices and hence FVA should be ignored.

### 2.4.3 Criticism on the Black and Scholes Argument

FVA has been the centre of intense debate between quantitative analysts, traders, treasurers and risk managers. One controversial aspect of including FVA is that the law of one price is violated, which is intolerable to some traditional quantitative analysts (Laughton & Vaisbrot, 2012). The banks funding policy is an internal aspect, prices would become subjective, reflecting different funding spreads of different banks (Hull & White, 2012). Nevertheless, many practitioners state this has been a key factor driving bid-ask spreads and claim there is no way back to the pre-crisis world of unique prices (Carver, 2012). As professor Damiano Brigo formulates it: 'In finance we’ve been trained to think there is a Platonic price, one lesson of this crisis is that this law of one price is gone' (Carver, 2012).

Furthermore, the Black and Scholes argument is less relevant since the global credit crisis (Castagna, 2012). Pre crisis, many financial institutions were able to borrow (close to) the risk-free rate and since the Black and Scholes argument hinges upon agents who are able to borrow at the risk-free rate, the Black and Scholes PDE was valid. Nowadays, in practice banks typically borrow at a higher rate than the risk-free rate. To illustrate this analytically, consider the classical Black and Scholes argument, which is a consequence of replicating a derivative contract \( P \) by a portfolio of the underlying \( S \) and bonds \( B \). Following Castagna (2012):

\[
P_t = \alpha_t S_t + \beta_t B_t. \tag{2.13}
\]

Where, \( \alpha_t \) equals \( \Delta_t \), the first derivative of the contract with respect to \( S \) and \( \beta_t \) is chosen as

\[
\beta_t = \frac{P_t - \Delta_t S_t}{B_t}. \tag{2.14}
\]

\( S \) is assumed to to pay no dividend, so that

\[
dP_t = \Delta_t dS_t + (P_t - \Delta_t S_t) \bar{r}_t dt, \tag{2.15}
\]

with \( \bar{r}_t = r_t 1_{\{\beta > 0\}} + r^F_t 1_{\{\beta < 0\}} \). We denote with \( r \) the risk-free rate and with \( r^F \) the funding rate the bank has to pay on its debt. In the Black and Scholes argument it is assumed that, \( r = r^F \) and Eq. 2.15 equals the standard Black and Scholes PDE. However, in an economy where a bank pays a higher interest rate than the risk-free rate when borrowing (\( \beta < 0 \)), the Black and Scholes PDE in Eq. 2.15 may contain the funding rate \( r^F \). Hence, always discounting by the same risk-free rate may produce incorrect results, since it may not produce risk-neutral values corresponding to the costs of the replication strategy. The Black and Scholes PDE can also be derived by discounting the expected pay-off of a derivative in a risk-neutral world at the risk-free rate of interest (i.e. the fundamental theorem of asset pricing). However, a necessary assumption to obtain the Black and Scholes PDE
is completeness, which means that every claim can be perfectly hedged. Nevertheless, in reality markets are incomplete, portfolios cannot be perfectly hedged (Laughton & Vaisbrot, 2012).

Another important assumption in the reasoning of Hull & White (2012) is that the effect of a new deal on the funding costs of a bank is linear (i.e. the linear funding feedback assumption). To illustrate this assumption, consider a company that is worth 1 billion and has a credit spread or funding spread of 100 basis points. Assume the company invests an additional 1 billion into a new project which is risk-free. The linear funding feedback assumption implies that the company is now worth 2 billion dollar and that its funding spread dropped to 50 basis point. Morini (2014) questions this assumption, claiming the market funding feedback is highly non-linear. Consider for example a firm in a distressed country that is very exposed to its sovereign. In the case that the credibility of the sovereign stays the same, undertaking risk-free projects would merely decrease the funding spread of the firms.

Lastly, many practitioners argue funding cost are real costs that can dramatically impact a banks profit and loss statement. Hence, many big investment banks already implemented FVA. However, there is still lots of controversy and all these bank calculate FVA in a different way (Cameron, 2014).

2.4.4 Double Counting and the Bond-CDS Basis

A very influential paper on the incorporation of FVA has been written by Morini & Prampolini (2010). In this paper it is showed that the important variable determining the cost of liquidity is neither the bond spread nor the credit default swap (CDS) spread, but the bond-CDS basis. In this subsection the findings of Morini & Prampolini (2010) are briefly discussed.

Morini & Prampolini (2010) use a simple modeling setting, considering a deal involving a borrower $B$ and a lender $L$, where $B$ commits to pay a fixed amount $K$ to $L$ at time $T$. Assume that party $X \in \{B, L\}$, has a recovery rate $R_X$. This means that in case of a default of party $X$, $R_X$ is the percentage of the exposure that is recovered to creditors. The risk-free rate is denoted by $r$ and is assumed to be deterministic. Furthermore, $X$ funds itself in the bond market and is the reference entity in the CDS market. More generally, the CDS spread $\pi_X$ is assumed to be deterministic and paid continuously. In Section 3.3.2 of Brigo et al. (2013) it is showed that in this case:

$$\pi_X = \lambda_X LGD_X. \quad (2.16)$$

Where, $\lambda_X$ denotes the deterministic default intensity and the loss given default $LGD_X$ equals $1-R_X$. Recovery is assumed to be zero ($LGD_X = 1$), thus $\pi_X = \lambda_X$. From the exponential distribution assumption for default time $\tau$, it follows that $\mathcal{P}(\tau_X > T) = e^{-\pi_X T}$ (see section 3.3 of Brigo et al. (2013)).

Like $\pi_X$, the funding spread $s_X$ is also assumed to be instantaneous and deterministic. The cost of funding is commonly measured in the secondary bond market as the spread over a risk-free rate. The difference between the funding spread and the CDS spread of a party $X$ is called the liquidity basis and is denoted by $\gamma_X$, hence $s_X = \pi_X + \gamma_X$. 

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First, the net present value (NPV) for the lender $V_L$ of the above deal is described without including funding cost. If $P$ denotes the premium paid by $L$ at inception, the NPV of the above deal equals

$$V_L = e^{-rT} K - CVA_L - P. \quad (2.17)$$

Where $CVA_L$ is given by

$$CVA_L = \mathbb{E}[e^{-rT} K 1_{\{\tau_B \leq T\}}] = e^{rT} K P(\tau_B \leq T) = e^{-rT} K [1 - e^{-\pi B T}]. \quad (2.18)$$

To make the value of the contract fair we equate Eq. (2.17) to zero. Therefore, we have $P = e^{-rT} K - CVA_L$. From the perspective of the borrower, the NPV of the deal is

$$V_B = -e^{-rT} K + DVA_B + P, \quad (2.19)$$

with $CVA_L = DVA_B$. To make the value of the contract fair we set $V_B$ to zero, thus $P = e^{-rT} K - DVA_B$. Therefore, price symmetry is satisfied $V_B = V_L = 0$ and both parties may agree on the premium of the deal:

$$P = e^{-rT} e^{-\pi B} K. \quad (2.20)$$

Obviously, funding costs are not implemented in the above derivation. While $L$ needs to finance the claim $P$ until the maturity of the deal at its funding spread $s_L$, party $B$ can reduce its funding by $P$. Therefore, $B$ has a funding benefit and party $L$ needs to pay its financing cost and thus has funding costs. Hence, party $L$ should reduce the value of the claim by its financing costs. Besides, we cannot assume both parties having negligible funding cost since we are dealing with possible default risk. To introduce liquidity in the valuation of the deal, Morini & Prampolini (2010) describe the problem of double counting. In this case, we implement liquidity costs by (only) changing the discount factor. Moreover, the value to the lender is

$$V_L = \mathbb{E}[e^{-(r+s_L)T} K 1_{\{\tau_B > T\}}] - P = \mathbb{E}[e^{-rT} e^{-\gamma B T} e^{-\pi L T} K 1_{\{\tau_B > T\}}] - P = e^{-rT} e^{-\gamma L T} e^{-\pi L T} K e^{-\pi B T} - P, \quad (2.21)$$

and the value to the borrower is

$$V_B = -\mathbb{E}[e^{-(r+s_B)T} K 1_{\{\tau_B > T\}}] + P = -\mathbb{E}[e^{-rT} e^{-\gamma B T} e^{-\pi B T} K 1_{\{\tau_B > T\}}] + P = e^{-rT} e^{-\gamma B T} e^{-2\pi B T} K + P. \quad (2.22)$$

To discuss this finding, we assume for simplicity that $s_L = 0$, so the lender $L$ is default-free and has no liquidity basis. On the other hand, the borrower $B$ may default, thus $s_B = \pi_B > 0$. In this case we obtain $P_L = e^{-rT} e^{-\pi B T} K$ and $P_B = e^{-rT} e^{-2\pi B T} K$, which is a remarkable finding. Firstly, the two parties disagree on the premium of this simple deal. Borrowers can account an immediate
profit in all transaction with CVA. And secondly, pricing this deal at fair value to the borrower would involve multiplying the NPV of $K$ twice with its survival probabilities, which is called the problem of double counting in Morini & Prampolini (2010). Both of these aspects belie years of market reality.

In order to solve this puzzle, Morini & Prampolini (2010) model the funding strategy explicitly. Following this approach, the deal is split up into two legs. From the lender’s perspective, the NPV of the ‘deal leg’ is given by

$$E[-P + e^{-rT} \Pi],$$

(2.23)

where $\Pi$ denotes the pay-off at $T$ with a potential default indicator. The other leg is called the ‘funding leg’ and NPV

$$E[P - e^{-rT} F],$$

(2.24)

where $F$ is the funding payment at $T$, also including a potential default indicator. Therefore, the total NPV equals

$$V_L = E[e^{-rT} \Pi - e^{-rT} F].$$

(2.25)

Morini & Prampolini (2010) make the assumption funding is made by issuing bonds and excess funds are used to reduce or avoid increasing the stock of bonds. Therefore, the outflow $F$ at $T$ is

$$Pe^{\gamma_L T} \mathbf{1}_{\{\tau_L > T\}}.$$  

(2.26)

In the ‘deal leg’, the lender inflow $\Pi$ at $T$ is $K \mathbf{1}_{\{\tau_B > T\}}$. Thus, the total pay-off at $T$ is

$$- Pe^{\gamma_L T} e^{\pi_L T} \mathbf{1}_{\{\tau_L > T\}} + K \mathbf{1}_{\{\tau_B > T\}}.$$  

(2.27)

Taking the discounted expectation of Eq. 2.27 yields

$$V_L = -Pe^{\gamma_L T} + Ke^{-rT} e^{-\pi_B T}.$$  

(2.28)

Analogously, it can be shown that the NPV of the deal for the borrower is

$$V_B = P e^{\gamma_B T} - Ke^{-rT} e^{-\pi_B T},$$  

(2.29)

where the double counting problem vanished. It can easily be shown that the break-even premium for the lender is $P_L = Ke^{-rT} e^{-\pi_B T} e^{-\gamma_L T}$ and for the borrower $P_B = Ke^{-rT} e^{-\pi_B T} e^{-\gamma_B T}$. To reach an agreement: $V_L \geq 0, V_B \geq 0$, has to be satisfied, this implies $P_L \geq P \geq P_B$. Thus an agreement can be found whenever, $Ke^{-rT} e^{-\pi_B T} e^{-\gamma_B T} \geq P \geq Ke^{-rT} e^{-\pi_B T} e^{-\gamma_B T}$, which holds when:

$$\gamma_B \geq \gamma_L.$$  

(2.30)

This shows that, in order to have a positive NPV for both counterparties, the funding cost that needs to be charged in this simplified transaction is just the liquidity basis. The lender’s funding cost contains a part that is associated with the lender’s probability of default. This part cancels out with the probability of default in the lender’s funding strategy, hence the only spread that contributes as a net funding cost to the lender is the liquidity basis.
Chapter 3

Methodology

3.1 Interest Rate Model: G2++

To model the evolution of the Euribor rate (i.e. the underlying risk factor) we use a Gaussian shifted two-factor process (hereafter G2++). This model is equivalent to the renowned Hull-White 2-factor short rate model \cite{HullWhite1994}, only parametrized differently. To introduce this model we will first show the single factor Hull-White process and discuss its limitations. Thereafter, we describe the G2++ model and the associated calibration to market instruments. For basic theory regarding interest rate modelling we refer to Chapters 3 and 4 of \cite{BrigoMercurio2006}.

3.1.1 Single Factor Hull-White Model

The single factor Hull-White model \cite{HullWhite1990} is the forerunner of the G2++ model and historically one of the most important interest rate models. The instantaneous short-rate $r(t)$ evolves according to the following stochastic differential equation:

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t),$$

where $a$ indicates the rate of mean reversion to time-varying mean $\theta(t)$, $\sigma$ is a volatility parameter and $W(t)$ a Brownian motion under the risk neutral measure $Q$. The model implies many convenient features such as closed-form bond prices and a Gaussian distribution for the short-rate $r$. Though, the model has an important disadvantage. To illustrate this disadvantage, consider the continuously compounded spot interest rate $R(t, T) = -\ln P(t, T) r(t)$, which depends through $P(t, T)$ on the dynamics of the short rate $r(t)$. Moreover, consider a pay-off depending on the joint distribution of $R(t, T_1)$ and $R(t, T_2)$, with $T_1, T_2 > t$ and $T_1 \neq T_2$. In the single factor Hull-White model the correlation between $R(t, T_1)$ and $R(t, T_2)$ is 1, while in reality these rates are not perfectly correlated. The model implies that shocks to the interest rate curve at time $t$ are conveyed identically for different maturities, which may be an unagreeable feature. Clearly, the pay-off of an IRS will depend on the correlation between rates of different maturities. To overcome this problem of perfect correlation we move to the G2++ model.
3.1.2 The G2++ Model

The G2++ model (Brigo & Mercurio, 2006) extends the single factor Hull-White model by assuming the following stochastic differential equation for short-rate $r(t)$:

$$dr(t) = x(t) + y(t) + \varphi(t), \quad r(0) = r_0,$$

$$dx(t) = -ax(t)dt + \sigma dW_1(t), \quad x(0) = 0,$$

$$dy(t) = -by(t)dt + \eta dW_2(t), \quad y(0) = 0.$$  (3.2, 3.3, 3.4)

Where, $r_0, a, b, \sigma, \eta \geq 0$. Furthermore, $W_1, W_2$ are two correlated Brownian motions under $Q$ with $dW_1(t)dW_2(t) = \rho dt$ and $\rho$ denoting the instantaneous correlation between these Brownian motions.

We use the G2++ model to model the instantaneous Euribor rates. The model enables analytical formulae for various interest rate derivatives e.g. bond prices and swaptions, implies a Gaussian distribution for short-rate $r$ and allows for a non-perfect correlation structure between rates of different maturities. A disadvantage of the G2++ model (and the single factor Hull-White model) is the possibility of negative rates. Even so, the probability of negative interest rates is small and as negative interest rates might occur in the current market environment we consider this acceptable.

3.1.3 Calibration of the G2++ Model

To calibrate the G2++ model we first choose $\varphi$ such that the model implied zero coupon bond prices $P(0,T)$ equals the market observed term structure of discount factors $P^M(0,T)$. In Section 4.2 of Brigo & Mercurio (2006) it is showed that the G2++ model is fitted to the market observed term structure of discount factor if and only if, for each $T$,

$$\varphi(T) = f^M(0,T) + \frac{\sigma^2}{2a^2} \left( 1 - e^{-aT} \right)^2 + \frac{\eta^2}{2b^2} \left( 1 - e^{-bT} \right)^2 + \rho \frac{\sigma \eta}{ab} \left( 1 - e^{-aT} \right) \left( 1 - e^{-bT} \right),$$  (3.5)

where $f^M(0,T) = \frac{-\partial \ln P^M(0,T)}{\partial T}$, i.e. the current market implied instantaneous forward rate for a maturity $T$. Thence, we choose $\varphi$ according to Eq. 3.5.

The model parameters $a, b, \sigma, \eta$ and $\rho$ are calibrated to market prices of at-the-money swaptions. A swaption is (typically) an option on an interest rate swap, i.e. a swaption holder has the right, but not the obligation to enter into an IRS at a pre-specified future date. A swaption is said to be at-the-money when the fixed rate of the swaption (i.e. the strike of the swaption) equals the current swap rate in the market. Given a strike rate $X$, strike date $T$, a nominal value $N$ and payment schedule $T = \{t_1, \ldots, t_n\}$, $t_1 > T$, the G2++ model implies a arbitrage-free price for European swaptions. In Brigo & Mercurio (2006) it is shown that this price, which we denote by $ES$, equals

$$ES(0, T, T, N, X, \omega) = NP(0, T)\omega \int_{-\infty}^{\infty} \left( 1 - \sum_{i=1}^{n} c_i A(T, t_i) e^{-B(a(T, t_i)x-B(b, T, t_i)y)} \right)^+ f(x, y)dydx,$$
where \( c_i = X \tau_i \) for \( i = 1, ..., n - 1 \) and \( c_n = 1 + X \tau_n, \omega = 1 (\omega = -1) \) for a payer (receiver) swaption, the density \( f \) of \((x(T), y(T))\) is given by

\[
f(x, y) = e^{-\frac{1}{2[(1-\rho^2_{xy})]}
\left[
\frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho_{xy}\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}
\right]}
\frac{2\pi \sigma_x \sigma_y \sqrt{1 - \rho_{xy}^2}}{\sigma_x^2 + \sigma_y^2},
\]

(3.6)

and

\[
\begin{align*}
\mu_x &= -M_x^T(0, T) \\
\mu_y &= -M_y^T(0, T) \\
\sigma_x &= \sigma \sqrt{\frac{1 - e^{-2aT}}{2a}} \\
\sigma_y &= \eta \sqrt{\frac{1 - e^{-2bT}}{2b}} \\
\rho_{xy} &= \frac{\rho \sigma \eta}{(a + b)\sigma_x \sigma_y} \left[1 - e^{-(a+b)T}\right].
\end{align*}
\]

In order to calibrate the model, we choose the model parameters \( a, b, \sigma, \eta \) and \( \rho \) such that the sum of the squared percentage differences between the model implied swaption volatilities and market implied swaption volatilities are minimized. The implied swaption volatilities are calculated using the Black formula for swaptions \([Brigo & Mercurio 2006]\).

### 3.2 Euribor-OIS Spread Model

Modelling the OIS rates with the G2++ model described in the previous section is troublesome, since the calibration would be inaccessible. For example, there are no overnight index swaptions available in the market. Instead of modelling the OIS rates themselves, OIS rates are obtained by modelling the Euribor-OIS spread \( s \). We have, \( s(t) = r(t) - c(t) \), where \( r(t) \) indicates the Euribor rate and \( c(t) \) the OIS rate at time \( t \). Theoretically, the Euribor-OIS spread should be a positive quantity as the OIS rate should be lower than the associated Euribor rate. To model such an always-positive process we adopt a CIR process, introduced by \([Cox et al. 1985]\), where we wish to simulate \( s(t) \) under the risk-neutral measure \( Q \). To do so, we first derive the Euribor-OIS spread dynamics under the real-world measure \( P \) and subsequently account for the market price of risk to obtain the dynamics under \( Q \). We assume that the stochastic differential equation for the Euribor-OIS spread under the real-world measure \( P \) is given by

\[
ds(t) = \tilde{\kappa}(\tilde{\mu} - s(t))dt + \tilde{\nu}\sqrt{s(t)}d\tilde{W}(t),
\]

(3.7)

where \( \tilde{W} \) is a standard Brownian motion under \( P \) and \( \tilde{\kappa}, \tilde{\mu}, \tilde{\nu} > 0 \). Moreover, \( \kappa \) corresponds to the speed of mean reversion, \( \mu \) to the mean and \( \nu \) is a volatility parameter. We impose \( 2\kappa\mu > \nu^2 \) to keep \( s(t) \) positive. Likewise, we assume that the dynamics under \( Q \) are given by

\[
ds(t) = \kappa(\mu - s(t))dt + \nu\sqrt{s(t)}dW(t),
\]

(3.8)
where $W$ is a standard Brownian motion under $Q$ and $\kappa, \mu, \nu > 0$. We adopt a similar methodology as in Cox et al. (1985), and set $\kappa = \tilde{\kappa} + \lambda, \mu = \tilde{\mu} + \lambda$ and $\nu = \tilde{\nu}$, where $\lambda$ denotes the market price of risk.

### 3.2.1 Calibration of the Euribor-OIS Spread Model

In order to calibrate the Euribor-OIS spread model, we first calibrate the model parameters $\tilde{\kappa}, \tilde{\mu}, \tilde{\nu}$ to historical market data of Eonia and Euribor spot rates. Consider the Euler discretization of Eq. (3.7):

$$s(t + \delta t) = s(t) + \tilde{\kappa}(\tilde{\mu} - s(t))\delta t + \tilde{\nu} \sqrt{s(t)}Z(t),$$

where $Z$ is a standard-normal random variable. Equivalently,

$$\frac{s(t + \delta t) - s(t)}{\sqrt{s(t)}} = \frac{\tilde{\kappa} \tilde{\mu} \delta t}{\sqrt{s(t)}} - \tilde{\kappa} \sqrt{s(t)}\delta t + \tilde{\nu} Z(t).$$

Note that the above equation can be seen as a linear regression model with dependent variable $s(t + \delta t) - s(t) \sqrt{s(t)}$ and explanatory variables $\frac{\delta t}{\sqrt{s(t)}}$ and $\delta t \sqrt{s(t)}$. Therefore, $\tilde{\kappa}, \tilde{\mu}, \tilde{\nu}$ can be estimated consistently by Ordinary Least Squares.

Thereafter, we estimate the market price of risk $\lambda$ by minimizing the sum of squared differences between a set of model implied bond prices and market implied bond prices. As in Hull (2009), the bond price $P_{CIR}(t, T)$ implied by the CIR model, is given by

$$P_{CIR}(t, T) = A(t, T)e^{-B(t, T)s(t)},$$

where,

$$A(t, T) = \frac{2h e^{\frac{1}{2}((\kappa + h)(T-t))}}{2h + (\kappa + h)(e^{h(T-t)} - 1)},$$

$$B(t, T) = \frac{2(e^{h(T-t)} - 1)}{2h + (\kappa + h)(e^{h(T-t)} - 1)}$$

and $h = \sqrt{\kappa^2 + 2\nu^2}$. Since we are modelling the spread between the two instantaneous interest rates $r(t)$ and $c(t)$, the relevant market implied bond prices are given by

$$P_{MKT}(t, T) = \mathbb{E}_Q[e^{-\int_t^T (r(u) - c(u))du}].$$

Note that theoretically, a bond price is just the risk neutral expectations of the relevant discount factors. Moreover, we can write Eq. (3.14) as the risk neutral expectation of a quotient of two stochastic discount factors

$$P_{MKT}(t, T) = \mathbb{E}_Q \left[ \frac{e^{-\int_t^T r(u)du}}{e^{-\int_t^T c(u)du}} \right] = \mathbb{E}_Q \left[ \frac{D_{eur}(t, T)}{D_{ois}(t, T)} \right],$$

where $D_{eur}$ and $D_{ois}$ are the stochastic discount factors of the instantaneous Euribor and OIS rate.
respectively. Equivalently, we can write Eq. 3.15 as

\[ P^{\text{MKT}}(t,T) = \frac{\mathbb{E}_Q[D_{\text{eur}}(t,T)]}{\mathbb{E}_Q[D_{\text{ois}}(t,T)]} + \text{cov}\left[D_{\text{eur}}(t,T), \frac{1}{D_{\text{ois}}(t,T)}\right]. \]  

(3.16)

Using historical data, we can estimate the latter covariance term, which we find to be negligible in magnitude. Nevertheless, we do include the term in the calculation. Note that we can view the former term of Eq. 3.16 as a quotient of two bond prices as \( P_{\text{eur}}(t,T) = \mathbb{E}_Q[D_{\text{eur}}(t,T)] \) and \( P_{\text{ois}}(t,T) = \mathbb{E}_Q[D_{\text{ois}}(t,T)] \), which are market observable.

To estimate the market price of risk, we vary \( \lambda \) to minimize the sum of squared differences between the current model implied bond prices \( P_{\text{CIR}}(0,T) \) and the current market implied bond prices \( P^{\text{MKT}}(0,T) \), for a set of different maturities. When we have estimated \( \lambda \) we can determine \( \kappa \) and \( \mu \) by \( \kappa = \tilde{\kappa} + \lambda \) and \( \mu = \frac{\tilde{\kappa} \tilde{\mu}}{\tilde{\kappa} + \lambda} \).

### 3.3 Credit Spread Model

#### 3.3.1 Stochastic Intensity

Possible default events are modelled by a stochastic intensity model called the CIR++ model. Stochastic intensity models describe default time \( \tau \) as the first jump time of a Cox process, that is a Poisson process with stochastic intensity. The basic idea behind these models is that default has an exogenous element, which is independent of market observables. Indeed, market data does not give complete information on the default process. Therefore, intensity \( \lambda_t \) is assumed to be time-varying and stochastic. More explicit, having not defaulted before time \( t \), the probability of defaulting in the next time instant \( dt \) is \( \lambda_t dt \). Thereby, we will often use the cumulated intensity \( \Lambda = \int_0^t \lambda(u)du \). This sort of default models are well-suited to model credit spreads and calibration to Credit Default Swap (CDS) data is well-to-do (Brigo & Mercurio, 2006).

To show why these models are well suited for credit spreads, suppose for simplicity deterministic intensity \( \lambda \). An peculiarity of Poisson processes is a result of the transformation of jump time \( \tau \) according to its own cumulated intensity \( \Lambda \) (Brigo & Mercurio, 2006). Hence, \( \Lambda(t) = \xi \sim \text{exponential standard random variable} \). Inverting this result yields \( \tau = \Lambda^{-1}(\xi) \). By recalling \( Q(\xi \leq x) = 1 - e^{-x} \), i.e. the cumulative distribution function of an exponential standard random variable, we have

\[ Q\{\tau > t\} = Q\{\Lambda(\tau) > \Lambda(t)\} = Q\{\xi > \Lambda(t)\} = e^{-\int_0^t \lambda(u)du}. \]  

(3.17)

Note that this last term looks very much alike a discount factor. Moreover, if we return to stochastic intensities again, the survival probability at time \( t \) becomes

\[ Q\{\tau > t\} = \mathbb{E}[e^{-\int_0^t \lambda(u)du}] \]  

(3.18)

Remarkably, this is just the price of a zero coupon bond \( P \) in an interest rate model with short rate \( r \) replaced by \( \lambda \). Hence, survival probabilities can be interpreted as zero coupon bonds and vice versa.
Consequently, short-rate models may be very useful in modelling intensities. Given the fact that intensities should be positive quantities, the CIR setting we have seen in Section 3.2 is the model we use to model intensities.

### 3.3.2 CDS Calibration

Credit Default Swaps are the market instruments from which market implied default probabilities are drawn. A CDS is an agreement in which the buyer of the CDS makes payments to the seller of the CDS, until the maturity date of the contract. In exchange, the buyer receives credit protection for a potential default of a third party. Thus, if the buyer has a credit exposure to this third party, the seller will reimburse the losses the buyer would suffer in case this third party defaults on (for example) a loan.

To strip default intensities from CDS quotes, we use the procedure described in Section 22.3 of [Brigo & Mercurio, 2006]. We assume intensity \( \gamma(t) \) to be deterministic and piecewise constant and define hazard function \( \Gamma(t) = \int_0^t \gamma(u) du \). Moreover, \( \gamma(t) = \gamma_i \) for \( t \in [T_{i-1}, T_i) \), where the \( T_i \)'s span the relevant maturities. In this setting \( \Gamma(t) = \int_0^t \gamma(u) du = \sum_{i=1}^{\beta(t)-1} (T_{i+1} - T_i) \gamma_i + (t - T_{\beta(t)-1}) \gamma_{\beta(t)} \), where \( \beta(t) \) is the index of the first \( T_i \) following \( t \). Likewise, set \( \Gamma_j = \int_{T_j}^{T_{j+1}} \gamma(s) ds = \sum_{i=1}^{j} (T_i - T_{i-1}) \gamma_i \).

The protection leg of a CDS can now be expressed as (Brigo & Mercurio, 2006):

\[
LGD \mathbb{E}[D(0, \tau) 1_{\{T_a < \tau < T_b\}} | \mathcal{F}] = LGD \int_0^\infty \mathbb{E}[D(0, u) 1_{\{T_a < u < T_b\}}] \mathbb{Q}(\tau \in [u, u + du])
\]

\[
= LGD \int_{T_a}^{T_b} \mathbb{E}[D(0, u)] \mathbb{Q}(\tau \in [u, u + du])
\]

\[
= LGD \int_{T_a}^{T_b} P(0, u) \gamma(u) \exp \left( - \int_0^u \gamma(s) ds \right) du
\]

\[
= LGD \sum_{i=a+1}^b \gamma_i \int_{T_{i-1}}^{T_i} \exp \left( - \Gamma_{i-1} - \gamma_i (u - T_{i-1}) \right) P(0, u) du. \quad (3.19)
\]

Where the CDS ensures credit protection in \([T_a, T_b]\) and the filtration \( \mathcal{F}_t \) represents all the observable market quantities up to time \( t \). Besides, the premium leg of a CDS can be derived by a similar computation. Under this formulation, it can be shown that the net present value of a CDS contract is

\[
CDS_{a,b}(t, R, LGD; \Gamma(\cdot)) = 1_{\{\tau > t\}} \left[ R \int_{T_a}^{T_b} P(t, u) (T_{\beta(u)} - u) e^{-(\Gamma(u) - \Gamma(t))} du + \sum_{i=a+1}^b P(t, T_i) R \alpha_i e^{-(\Gamma(T_i) - \Gamma(t))} + LGD \int_{T_a}^{T_b} P(t, u) e^{-(\Gamma(u) - \Gamma(t))} du \right].
\]
Where the buyer of the CDS pays rate $R$ at $T_{a+1},...,T_b$ to the protection seller and $\alpha_i = T_i - T_{i-1}$.
In our case of piecewise constant intensity $\gamma(t)$, we have

$$
CDS_{a,b}(t, R, LGD; \Gamma(\cdot)) = R \sum_{i=a+1}^{b} \gamma_i \int_{T_{i-1}}^{T_i} \exp(-\Gamma_{i-1} - \gamma_i(u - T_{i-1})) P(0, u)(u - T_i)du
+ R \sum_{i=a+1}^{b} P(0, T_i)\alpha_i e^{-\Gamma(T_i)}
- LGD \sum_{i=a+1}^{b} \gamma_i \int_{T_{i-1}}^{T_i} \exp(-\Gamma_{i-1} - \gamma_i(u - T_{i-1})) P(0, u)du.
$$

In order to perform the case study in Chapter 4, we obtained market CDS spreads from Bloomberg on April 30, 2014 for our entities (Deutsche Bank AG and Koninklijke Ahold NV). Both CDS’s involve quarterly payments, hence we set $\alpha_i = 0$. Furthermore, we have $T_b = 0.5y, 1y, 2y, 3y, 4y, 5y, 7y, 10y$ and $T_a = 0$ for both CDS quotes. To find $\gamma_i$, $i \in \{1, 2, ..., 40\}$, we solve

- $CDS_{0,0.5y}(0, R_{0,0.5y}^{MKT}, LGD, \gamma_1 = \gamma_2 = \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot = \gamma^1) = 0$;
- $CDS_{0,1y}(0, R_{0,1y}^{MKT}, LGD; \gamma^1, \gamma_3 = \gamma_4 = \cdot \cdot \cdot \gamma^2) = 0$;
- $CDS_{0,2y}(0, R_{0,2y}^{MKT}, LGD; \gamma^2, \gamma_5 = \gamma_6 = \cdot \cdot \cdot = \gamma^3) = 0$;

and so so on. Each time obtaining the next relevant market implied default intensities. Note that we are now able to construct the market implied survival function $e^{-\Gamma(t)}$ up to 10 years for both entities.

### 3.3.3 CIR++ Intensity Model

In the CIR++ intensity model the dynamics of (stochastic) intensity $\lambda_t$ is given by:

$$
\lambda_t = y_t + \psi(t; \beta), \quad t \geq 0.
$$

(3.20)

Here $\psi$ indicates a deterministic function, which can be used to fit the term structure of credit spreads. Note that the CIR model can be obtained from the CIR++ model by equating $\psi(t; \beta) = 0$.

The parameter vector $\beta = (\kappa, \mu, \nu, y_0)$, with $\kappa, \mu, \nu, y_0$ positive constants, drives the dynamics of $y$ as

$$
dy_t = \kappa(\mu - y_t)dt + \nu \sqrt{y_t}dZ_t,
$$

(3.21)

where $Z$ is a standard Brownian motion process under the risk neutral measure. $Z$ can be seen as the stochastic shock in our intensity dynamics. We impose $2\kappa\mu > \nu^2$ in order to keep $\lambda$ positive \cite{BrigoMercurio2006}. 

3.3.4 Exact Fit to Spread Curve

The CIR++ model extends the CIR model by allowing for an exact fit to market implied credit spreads. Moreover, \( \psi(t; \beta) \) is selected such that the model is calibrated to the market implied hazard function \( \Gamma_{mkt} \), i.e. to the CDS data. The market implied hazard function and associated market implied intensity \( \gamma_{mkt} \) are obtained according to the method described in Subsection 3.3.2.

As already mentioned in Subsection 3.3.1, survival probabilities can be interpreted as zero coupon bonds. Hence, the survival probabilities in the CIR++ intensity model will be given by the CIR++ short-rate model bond price formula. Define the integrated quantities \( Y(t) = \int_0^t y_s ds \) and \( \Psi(t; \beta) = \int_0^t \psi_s ds \). The model implied survival probability reads

\[
Q(\tau > t)_{model} = \mathbb{E}[e^{-\Lambda(t)}] = \mathbb{E}[\exp(-\Psi(t, \beta) - Y(t))],
\]

whereas the market implied survival probability is \( Q(\tau > t)_{market} = e^{-\Gamma_{mkt}} \). Hence, in order to have \( Q(\tau > t)_{market} = Q(\tau > t)_{model} \), we just need to make sure that

\[
Q(\tau > t)_{market} = \mathbb{E}[\exp(-\Psi(t, \beta) - Y(t))].
\]

Thus, we need to set \( \Psi(t, \beta) \) according to

\[
\Psi(t, \beta) = \Gamma_{mkt}(t) + \ln(\mathbb{E}[e^{-Y(t)}]) = \Gamma_{mkt}(t) + \ln(P_{CIR}(0, t, y_0; \beta)),
\]

in order to calibrate the CIR++ model to the market implied survival function \( \Gamma_{mkt} \). Note that in the above formulation independence between \( r \) and \( \tau \) is assumed. Though, in Brigo & Mercurio (2006) it is showed that the impact of a correlation between \( r \) and \( \tau \) is typically negligible on CDS’s.

3.3.5 Calibrating Model Parameters

Now that we have we fitted the model to CDS data through \( \psi \), the remaining parameters \( \beta \) can be calibrated to other securities. In risk-neutral models market option data is often used for this purpose. For example, swaption prices are used to calibrate the G2++ model parameters. Likewise, in the case of the CIR++ model, options on CDS contracts could be useful. However, bid-ask spreads for single name CDS option are currently large, indicating a illiquid market. Therefore, calibrating intensity models to CDS options will become more interesting in the future as liquidity might improve. In this thesis, we choose reasonable values for \( \beta \) based on the estimates of Brigo & Mercurio (2006) which uses a hypothetical implied volatility surface of CDS options. Nevertheless, we analyse the \( \mu \) and \( \nu \) on BCVA and FVA. Calibrating the CIR++ model parameters to real market data is left for future research.
3.4 Modelling Bilateral Credit Value Adjustment

We discussed the intuition behind incorporating BCVA in Section 2.3 and showed that BCVA makes the valuation of bilateral counterparty credit risk symmetric. Moreover, BCVA has to be subtracted from the value of the interest rate swap assuming no default risk. In this section we will describe the procedure regarding the calculation of BCVA, which involves both Monte Carlo integration and Euler discretization. We will discuss these methods in respectively Subsection 3.4.1 and Subsection 3.4.2. Thereafter, we derive the formula for BCVA based on Gregory (2013) and finish with discussing Wrong-way Risk.

3.4.1 Monte Carlo Integration

Monte Carlo integration is a widely used statistical method, which is helpful for obtaining numerical solutions to problems which may be too complex to solve analytically. For the calculation of BCVA, we need expectations which are inaccessible, hence we use Monte Carlo integration to analyse these expectations. Consider for example a vector of random variables \( X = \{X_1, ..., X_K\} \), with joint probability density functions \( f(x_1, ..., x_K) \), and suppose we want to compute the expectation \( \mathbb{E}[g(X)] \). Monte Carlo integration involves sampling \( X \) for \( i = 1, 2, ..., N \) from the joint distribution \( f(x_1, ..., x_K) \). Hereafter, the demanded expectation can be approximated by

\[
\mathbb{E}[g(X)] = \frac{1}{N} \sum_{n=1}^{N} g(x_{1,i}, ..., x_{K,i}).
\]  
(3.25)

Irrespective of the number of random variables, it can be showed that the estimation error decreases as \( \frac{1}{\sqrt{N}} \) (Press et al., 2002). Hence, increasing the sample by a factor 100, will reduce the estimation error by 90%.

3.4.2 Euler Discretization

We defined continuous-time stochastic processes for both interest rates and credit spreads. Yet, to perform simulations regarding these processes a discretization scheme is required. Euler discretization is a method to approximate solutions of stochastic differential equations numerically, thence we use Euler discretization to simulate our stochastic processes. To illustrate, suppose we want to discretize a stochastic differential equation of the form

\[
dy(t) = \theta(y(t), t)dt + \sigma(y(t), t)dW_t.
\]  
(3.26)

At time \( t + \delta t \) the value of \( y \) is

\[
y(t + \delta t) = y(t) + \int_{t}^{t+\delta t} \theta(y(s), s)ds + \int_{t}^{t+\delta t} \sigma(y(s), s)dW_s.
\]  
(3.27)
The two integrals can be approximated using the left endpoint approximation:

\[
\int_{t}^{t+\delta t} \theta(y(s), s) ds \approx \theta(y(t), t) \delta t
\]  

(3.28)

and

\[
\int_{t}^{t+\delta t} \sigma(y(s), s) dW_s \approx \sigma(y(t), t) (W_{t+\delta t} - W_t) = \sigma(y(t), t) \sqrt{\delta t} Z,
\]

(3.29)

where \( Z \) is standard normally distributed. Note that we used \( W_{t+\delta t} - W_t \sim N(0, \delta t) \) by definition, hence \( W_{t+\delta t} - W_t \) and \( \sqrt{\delta t} Z \) have the same distribution. Therefore, the Euler discretization of Eq. \ref{eq:3.27} is

\[
y(t + \delta t) \approx y(t) + \theta(y(t), t) \delta t + \sigma(y(t), t) \sqrt{\delta t} Z.
\]

(3.30)

So Euler discretization enables us to simulate different scenarios for all our stochastic risk factors up to respective time horizons. Besides, in case of correlated risk factors (e.g. correlated interest rates and credit spreads), one can correlate the associated stochastic differential equations by correlating their underlying Brownian motions. In this case one needs to sample correlated standard normal variables and use these samples as input for the corresponding discretization scheme.

Since the Euler scheme provides an approximation of the exact solution of the stochastic differential equation, we may question how good this approximation is. Orders of convergence are useful in determining the accuracy of different discretization schemes. Recall from Kloeden & Platen (1992) that if a numerical scheme is convergent with order \( \gamma \), then decreasing the step size \( \delta t \) with a factor \( k \) implies that the approximation error will decrease by a factor \( k^{\gamma} \). Hence, a large \( \gamma \) implies that the discretization error converges faster to zero. As an IRS does not have a path depended pay off (such as for example an Asian option), we just want to ensure that \( \hat{y}(t) \) (i.e. \( y(t) \) approximated by the Euler discretization scheme) is close to the exact value of \( y(t) \), hence we care about the order of weak convergence. The Euler scheme is typically weakly convergent with order 1 (Kloeden & Platen, 1992), thus if we make the step size 10 times smaller, then the discretization error will also become 10 times smaller. Note that better rates of convergence can be achieved by implementing more sophisticated discretization schemes. For example the Milstein scheme may be implemented, though we leave the implementation of more sophisticated discretization schemes as a possible extension of this thesis.

### 3.4.3 Positive-semidefinite Correlation Matrix: Hypersphere Decomposition

Simulating the (correlated) stochastic processes used to calculate BCVA and FVA involves sampling random shocks from a multivariate normal distribution. A requirement for the covariance matrix of this multivariate normal distribution is positive-semidefiniteness. Unfortunately, performing the methodologies described, we may obtain a covariance matrix which fails to be positive-semidefinite. For example, in cases of strong wrong-way risk the covariance matrix may fail to meet the requirement of positive-semidefiniteness. To overcome this undesirable aspect, we use Hypersphere decomposition described by Rebonato & Jäckel (1999). This method transforms the original matrix to a similar positive-semidefinite one, insuring unit diagonals and symmetry, which is required for simulating Brownian motions. Rebonato & Jäckel (1999) also show that Hypersphere decomposition almost
always leads to very small differences between the original and transformed matrix.

### 3.4.4 The Equation for Bilateral Credit Value Adjustment

Consider a transaction involving two default risky entities: a bank \( bnk \) and a corporate \( cpt \). The NPV of the transaction at time \( t \) with maturity date \( T \) (without taking into account corporate default risk) is \( V(t, T) \) and we wish to find \( \bar{V}(t, T) \), the adjusted value of the derivative, subject to corporate default risk. Besides, the \( BCVA(t, T) \) is the difference between \( V(t, T) \) and \( V(t, T) \). We will address valuations from the perspective of the bank, hence cash flows received by \( bnk \) (and paid by \( cpt \)) will be positive whereas cash flows paid by \( bnk \) (and received by \( cpt \)) will be negative. Furthermore, \( \tau_{bnk} \) and \( \tau_{cpt} \) indicate the default times of respectively \( bnk \) and \( cpt \). In the same way, \( REC_{bnk} \) and \( REC_{cpt} \) denote their recovery fractions (i.e. the percentage of the claim that is recovered in case the associated entity defaults). Lastly, denote the first-to-default time as \( \tau^1 = \min(\tau_{bnk}, \tau_{cpt}) \). To find \( \bar{V}(t, T) \), we may distinguish three cases:

1. Neither bank nor corporate defaults before \( T \),
2. The corporate defaults first and before \( T \),
3. The bank defaults first and before \( T \).

In the first case, the corresponding (risky) payoff is

\[
1_{\{\tau^1 > T\}} V(t, T),
\]

where \( 1_{\{\tau^1 > T\}} \) equals 1 if \( \tau^1 > T \) and 0 otherwise. In the second case, the cash flows paid up to the default time are paid plus the payoff at default. Hence, the payoff equals

\[
1_{\{\tau_{cpt} \leq T\}} V(t, \tau_{cpt}) + 1_{\{\tau^1 \leq T\}} 1_{\{\tau_{cpt} = \tau^1\}} (REC_{cpt} V(\tau^1, T)^+ + V(\tau^1, T)^-),
\]

where \( V^- = \min(V, 0) \) and \( V^+ = \max(V, 0) \). Note that in this case \( bnk \) pays the risk-free value of the derivative to \( cpt \) if it is negative, i.e. when \( bnk \) is a debtor and \( cpt \) is a creditor. On the other hand, when the risk-free value of the derivative is positive at \( \tau_{cpt} \), \( bnk \) only receives a recovery fraction of the risk-free value of the derivative from the defaulted corporate \( cpt \). Likewise, the payoff in the third case equals

\[
1_{\{\tau_{bnk} \leq T\}} V(t, \tau_{bnk}) + 1_{\{\tau^1 \leq T\}} 1_{\{\tau_{bnk} = \tau^1\}} (REC_{bnk} V(\tau^1, T)^- + V(\tau^1, T)^+).
\]

By summing the above payoffs the adjusted value of the derivative, subject to corporate default risk is

\[
\bar{V}(t, T) = \mathbb{E}[1_{\{\tau^1 > T\}} V(t, T)
+ 1_{\{\tau^1 \leq T\}} V(t, \tau^1)
+ 1_{\{\tau \leq T\}} 1_{\{\tau_{cpt} = \tau^1\}} (REC_{cpt} V(\tau^1, T)^+ + V(\tau^1, T)^-)
+ 1_{\{\tau \leq T\}} 1_{\{\tau_{bnk} = \tau^1\}} (REC_{bnk} V(\tau^1, T)^- + V(\tau^1, T)^+)],
\]

27
where $\mathbb{E}^Q$ denotes the expectation under the risk-neutral probability measure. The above expression can be written as
\[
\bar{V}(t, T) = \mathbb{E}^Q[1_{\{\tau^1 \leq T\}}V(t, T) + 1_{\{\tau^1 > T\}}V(\tau^1, T) + 1_{\{\tau^1 \leq T\}}1_{\{\tau_{cpt} = \tau^1\}}(1 - REC_{cpt})V(\tau^1, T)^+ + 1_{\{\tau^1 \leq T\}}1_{\{\tau_{bnk} = \tau^1\}}(1 - REC_{bnk})V(\tau^1, T)^- + 1_{\{\tau^1 \leq T\}}1_{\{\tau_{bnk} = \tau^1\}}(1 - REC_{bnk})V(\tau^1, T)^+ - V(\tau^1, T)^-].
\]

Thence, we can write
\[
\bar{V}(t, T) = V(t, T) - \mathbb{E}^Q[1_{\{\tau^1 \leq T\}}1_{\{\tau_{cpt} = \tau^1\}}(1 - REC_{cpt})V(\tau^1, T)^+ + 1_{\{\tau^1 \leq T\}}1_{\{\tau_{bnk} = \tau^1\}}(1 - REC_{bnk})V(\tau^1, T)^-].
\]

Recall from Subsection 2.3.2 that $\bar{V} = V + BCVA$ and notice that the latter term in the above equation expresses BCVA. Thus, the equation for BCVA is
\[
BCVA(t, T) = -\mathbb{E}^Q[1_{\{\tau^1 \leq T\}}1_{\{\tau_{cpt} = \tau^1\}}(1 - REC_{cpt})V(\tau^1, T)^+] + 1_{\{\tau^1 \leq T\}}1_{\{\tau_{bnk} = \tau^1\}}(1 - REC_{bnk})V(\tau^1, T)^-].
\] (3.34)

Throughout this thesis, we assume deterministic recovery fractions and no simultaneous defaults. Hence,
\[
BCVA(t, T) = -(1 - REC_{cpt})\mathbb{E}^Q[1_{\{\tau^1 \leq T\}}1_{\{\tau_{cpt} = \tau^1\}}V(\tau^1, T)^+] - (1 - REC_{bnk})\mathbb{E}^Q[1_{\{\tau^1 \leq T\}}1_{\{\tau_{bnk} = \tau^1\}}V(\tau^1, T)^-].
\] (3.35)

To simplify the above expression furthermore, we need to make assumptions regarding the dependence structure of default events and exposure. When credit quality and exposure are assumed to be independent, the terms in Eq. (3.35) can be split up in different expectations, which can be multiplied to calculate BCVA. In this case, we obtain,
\[
BCVA(t, T) = -(1 - REC_{cpt})\mathbb{E}^Q[1_{\{\tau^1 \leq T\}}1_{\{\tau_{cpt} = \tau^1\}}]\mathbb{E}^Q[V(\tau^1, T)^+] - (1 - REC_{bnk})\mathbb{E}^Q[1_{\{\tau^1 \leq T\}}1_{\{\tau_{bnk} = \tau^1\}}]\mathbb{E}^Q[V(\tau^1, T)^-].
\] (3.36)

In general, we will refer to Expected Exposure as $EE(t, T) = \mathbb{E}^Q[V(t, T)^+]$, whilst Negative Expected Exposure is given by $NEE(t, T) = \mathbb{E}^Q[V(t, T)^-]$. Besides, the risk-neutral survival probability of entity $i$ in the interval $[t, T]$ is given by $S(t, T)_i = \mathbb{E}^Q[\tau_i > T]$. Note that the risk-neutral default probability in this interval is $1 - S(t, T)$. Furthermore, recall from Subsection 2.3.2 that $BCVA = DVA - CVA$. Indeed, as in [Gregory, 2013], in case of no wrong-way risk (which is
discussed in the next subsection), CVA is given by

\[
CVA(t, T) = (1 - REC_{cpt}) E^Q [1_{\{\tau^1 \leq T\}} 1_{\{\tau_{cpt} = \tau^1\}}] E^Q [V(\tau^1, T)^+],
\]

(3.37)

and DVA is given by

\[
DVA(t, T) = -(1 - REC_{bnk}) E^Q [1_{\{\tau^1 \leq T\}} 1_{\{\tau_{bnk} = \tau^1\}}] E^Q [V(\tau^1, T)^-].
\]

3.4.5 Wrong- and Right-way Risk

Wrong-way risk and right-way risk are phrases that are generally used to indicate dependence between exposure and counterparty credit quality \(\text{[Gregory, 2013]}\). In case this dependence is unpropitious, we speak of wrong-way risk. Whereas, in the case of a propitious dependence structure between exposure and counterparty credit quality, we speak of right-way risk. For interest rate swaps, one should consider a relation between interest rates and counterparty credit quality. High interest rates might cause (leveraged) firms to default, while low interest rates could indicate a recession where default events are more common. We analyse this effect by correlating the short rate factors of the G2++ model with the intensity process embedded in the CIR++ model. Moreover, we allow for an instantaneous correlation between the associated driving Brownian motions \(W_1, W_2\) and \(Z\) as in \(\text{Brigo & Mercurio, 2006}\), hence \(d\langle W_i, Z \rangle_t = \rho_i dt, i \in \{1, 2\}\). In this case, the resulting correlation between interest rates and credit spreads becomes

\[
\tilde{\rho}_i = \frac{d\langle r, \lambda \rangle_t}{\sqrt{d\langle r, r \rangle_t d\langle \lambda, \lambda \rangle_t}} = \frac{\sigma \rho_1 + \eta \rho_2}{\sqrt{\sigma^2 + \eta^2 + 2 \sigma \eta \rho_{12}}},
\]

(3.38)

where \(\rho_{12}\) denotes the instantaneous correlation between \(W_1\) and \(W_2\). To limit the number of free parameters we always set \(\rho_1 = \rho_2\). Finally, by varying \(\tilde{\rho}_i\) we may allow for wrong- and right-way risk.

3.5 Modelling Funding Value Adjustment

The occurrence of and the reasons why there is so much dissension surrounding FVA are discussed in Section 2.4. As a matter of fact, all banks that have incorporated FVA calculate it in different ways \(\text{[Cameron, 2014]}\). Meanwhile, some banks choose to ignore FVA, while other banks just do not know how to handle FVA. It may be clear that there is a great need for a market practice regarding the implementation of FVA. In this section we propose and motivate a method to implement FVA for an uncollaterized plain vanilla interest rate swap. We assume stochastic net funding cost, modelled by a CIR process. To avoid the problem of double counting we define net funding cost as the Bond-CDS basis plus a fixed premium representing the internal treasury’s lending rate. In order to describe these aspects, we use the same setting and notation as in Subsection 3.4.4.
3.5.1 The Equation for Funding Value Adjustment

The purpose of the incorporation of FVA is to include the funding cost related to the ‘production’ of a derivative’s transaction, i.e. the cost of funding the corresponding hedge over the maturity of the deal. Consider the bank $bnk$ and the corporate $cpt$ enter in an uncollaterlized plain vanilla interest rate swap. The bank perfectly hedges the uncollaterlized transaction by entering in a secured trans- action with an underlying CSA in the interbank market. We name the counterparty in this ‘hedge deal’ Bank B. Note that this is a typical situation, as the corporate generally requests the bank for an uncollaterlized transaction and not vice versa. As a matter of fact, the operational implications of CSA’s are often unmanageable for corporates (Cameron, 2014). Therefore, we do not consider a bilateral FVA, as proposed by Gunnesson & Moreles (2014). To limit complexity, we suppose the CSA specifies that cash is the only valid collateral and that the Euro is the only valid currency in which this cash can be posted. Besides, we assume that the collateral rate is the Eonia rate. Moreover, we suppose that in order to trade collateral the bank’s trading desk needs to borrow funds from the bank’s treasury desk. The associated internal treasury’s lending rate is the cost of funding for the bank’s trading desk and includes (for example) internal labor, system and administration costs. Lastly, we assume that the bank’s treasury desk funds itself by issuing bonds in the (money) market.

Situations in which exposure is positive will cause funding costs as the bank does not receive collateral from the corporate and must post collateral on the margin account associated with the hedged IRS. Figure 3.1 visualizes such a situation.

![Figure 3.1: Entities bnk and cpt entered in unsecured a derivative contract, which has positive value for bnk ($V > 0$). The value of the contract is negative in the secured hedge deal with Bank B ($V < 0$). This situation leads to funding costs for bnk.](image)
On the contrary, when exposure is negative, funding benefits will arise as the bank receives collateral from the margin account of the secured IRS (see Figure 3.2 for an envision of this situation).

Figure 3.2: Entities bnk and cpt entered in unsecured a derivative contract, which has negative value for bnk \((V < 0)\). The value of the contract is positive in the secured hedge deal with Bank B \((V > 0)\). This situation leads to a funding benefit for bnk.

Therefore, the relevant part of the positive exposure that should account for the FVA is the positive exposure plus the negative exposure (which is negative by definition). However, when the bank bnk or the corporate cpt defaults before the maturity of the deal, both future funding cost and benefits will vanish. We define the value of the derivative including BCVA and FVA as \(\bar{V} = V + BCVA + FVA\). Therefore, FVA can be given by

\[
FVA(t, T) = -\mathbb{E}^Q \left[ \int_t^T 1_{\{s > t\}} D(t, s) \gamma(s) (V(s, T)^+ + V(s, T)^-) ds \right],
\]

where \(\gamma(t)\) denotes the net funding cost at time \(t\).

As explained in Subsection 2.4.4, a possible choice for \(\gamma(t)\) is the Bond-CDS basis. The advantage of this choice is that it is market observable, so that entities can recognize the net funding cost of their counterparties. However, the problem of this choice is that the risk-neutral expectation of the Bond-CDS basis can invert, leading to inconsistent net funding cost \(\text{Cameron} (2014)\). Theoretically, a bond yield can be divided in a risk-free part, an entity specific credit spread and a liquidity premium \(\text{Morini & Prampolini} (2010)\). Thus, an entity’s credit spread should be less-than-or-equal to the bond spread. However, as already mentioned, under the risk-neutral measure the Bond-CDS basis may invert. Possible explanations for this include that CDS’s have become favorable securities for banks as this will reduce additional capital requirement for counterparty credit risk under the Third Basel Accord. Therefore, we question whether the Bond-CDS basis on itself is a representative
quantity of a bank’s funding cost. In fact, internal lending rates also affect funding cost. From a bank’s internal perspective the treasury desk typically lends money to the trading desks. Hence, the bank’s treasury desk acquires funds in the market, while the trading desks obtains funds from the treasury desk. The internal treasury’s lending rate is the cost of funding for the derivatives desk. For example labour, system and administration costs may be included in the internal treasury’s lending rate. Obviously, this rate affects FVA, but as it is not market observable other entities might not recognize a FVA based on this rate.

In order to model FVA we take the internal treasury’s lending rate $F$ fixed and specify the net funding cost at time $t$ as

$$\gamma(t) = b(t) - c(t) - \pi(t) + F,$$  \hspace{1cm} (3.40)

where $b$ denotes the banks bond yield, $c$ the collateral rate and $\pi$ the banks credit spread. Note that the credit spread $\pi$ enters the equation to avoid double counting.

### 3.5.2 Net Funding Cost Model: CIR Process

In Section 3.2 we described how a CIR process can be used to model the Euribor-OIS spread. To model the net funding cost $\gamma(t)$ we use a similar approach. This approach is suitable in the sense that net funding cost are assured to stay positive. Moreover, net funding costs are assumed to be time-varying and the model can be calibrated in a convenient way to market data. We assume the net funding cost $\gamma(t)$ evolve according to the CIR process

$$d\gamma(t) = \tilde{\kappa}(\tilde{\mu} - \gamma(t))dt + \tilde{\nu}\sqrt{\gamma(t)}d\tilde{W}(t),$$  \hspace{1cm} (3.41)

where $\tilde{W}$ is a standard Brownian motion under $P$ and $\tilde{\kappa}, \tilde{\mu}, \tilde{\nu} > 0$. Moreover, $\kappa$ corresponds to the speed of mean reversion, $\mu$ to the mean and $\nu$ is a volatility parameter. We impose $2\kappa\mu > \nu^2$ to keep $\gamma(t)$ positive. Likewise, we assume that the dynamics under $Q$ are given by

$$d\gamma(t) = \kappa(\mu - \gamma(t))dt + \nu\sqrt{\gamma(t)}dW(t),$$  \hspace{1cm} (3.42)

where $W$ a standard Brownian motion under $Q$ and $\kappa, \mu, \nu > 0$. We adopt a similar methodology as in Section 3.2 and set $\kappa = \tilde{\kappa} + \lambda$, $\mu = \frac{(\tilde{\kappa}\tilde{\mu})}{\tilde{\kappa} + \lambda}$ and $\nu = \tilde{\nu}$, where $\lambda$ denotes the market price of risk.

### 3.5.3 Calibration of the Net Funding Cost Model

In order to calibrate the net funding cost model, we first calibrate the model parameters $\tilde{\kappa}, \tilde{\mu}, \tilde{\nu}$ to historical market data of bond yields, Eonia spot rates and credit spreads. Consider the Euler discretization of Eq. 3.41

$$\gamma(t + \delta t) = \gamma(t) + \tilde{\kappa}(\tilde{\mu} - \gamma(t))\delta t + \tilde{\nu}\sqrt{\gamma(t)}Z(t),$$  \hspace{1cm} (3.43)
where $Z$ a standard-normal random variable. Equivalently,

$$
\frac{\gamma(t + \delta t) - \gamma(t)}{\sqrt{\gamma(t)}} = \tilde{\kappa} \mu \delta t \sqrt{\gamma(t)} - \tilde{\kappa} \sqrt{\gamma(t)} \delta t + \tilde{\nu} Z(t).
$$

Note that the above equation can be seen as a linear regression model with dependent variable $\frac{\gamma(t + \delta t) - \gamma(t)}{\sqrt{\gamma(t)}}$ and explanatory variables $\frac{\delta t}{\sqrt{\gamma(t)}}$ and $\delta t \sqrt{\gamma(t)}$. Therefore, $\tilde{\kappa} \mu$, $\tilde{\kappa}$ and $\tilde{\nu}$ can be estimated consistently by Ordinary Least Squares.

Thereafter, we estimate the market price of risk $\lambda$ by minimizing the sum of squared differences between a set of model implied bond prices and market implied bond prices. The model implied bond price $P_{CIR}(t, T)$ model, is given by Eq. 3.11 with $s(t)$ replaced by $\gamma(t)$. The relevant market implied bond price is given by

$$
P_{MKT}(t, T) = E_Q\left[e^{-\int_t^T (b(u) - c(u) - \pi(u) + F) du}\right].
$$

Note that theoretically, a bond price is just the risk-neutral expectation of the relevant discount factors. We assumed that the internal treasury’s lending rate $F$ is fixed, thus independent of the credit spread $\pi$. Nevertheless, we want to note that there might be a relation between internal lending rates and credit spreads, as the treasury desk of a distressed bank might ask higher interest from the relevant derivative desks. However, this aspect is beyond the scope of this thesis, hence we leave it for future research. Furthermore, we can rewrite Eq. 3.45 as

$$
P_{MKT}(t, T) = e^{-F(T-t)} E_Q\left[\frac{e^{-\int_t^T b(u) - c(u) du}}{e^{-\int_t^T \pi(u) du}}\right] + \text{cov}\left[e^{-\int_t^T b(u) - c(u) du}, \frac{1}{e^{-\int_t^T \pi(u) du}}\right].
$$

Using historical data, we can estimate the latter covariance term. The two risk-neutral expectations in the former term can be calculated using current bond yields, Eonia spot rates and credit spread data. In more detail, by definition the zero coupon bond curve at time $t$ is $E_Q\left[\int_t^T b(u) du\right]$, the Eonia yield curve at time $t$ equals $E_Q\left[\int_t^T c(u) du\right]$ and the market CDS curve at time $t$ is $E_Q\left[\int_t^T \pi(u) du\right]$, where $T$ corresponds to the associated (market observable) maturities.

To estimate the market price of risk, we vary $\lambda$ to minimize the sum of squares differences between the current model implied bond prices $P_{CIR}(0, T)$ and the current market implied bond prices $P_{MKT}(0, T)$, for a set of different maturities. After estimating $\lambda$ we can determine $\kappa$ and $\mu$ by $\kappa = \tilde{\kappa} + \lambda$ and $\mu = \frac{(\tilde{\kappa} \mu)}{\tilde{\kappa} + \lambda}$. Hence, Monte Carlo integration can be used to calculate the expected net funding cost under the risk-neutral measure, which is needed to calculate FVA.
Chapter 4

A Case Study

To analyze FVA in the fair valuation of interest rate swaps, we examine a common situation, where a bank and a corporate enter into a plain vanilla interest rate swap. Moreover, we want to adjust the price of this swap for counterparty credit risk and funding risk by calculating BCVA and FVA. We question what the total value adjustment (TVA) should be, where \( TVA = BCVA + FVA \). Hence, by applying the described methodology we calculate BCVA and FVA. Thereby, we perform an impact analysis as the calculations of BCVA and FVA are highly model dependent. Hence, we analyse the impact of wrong-way risk, credit spread levels and credit spread volatilities on BCVA and FVA.

4.1 Case Description

Suppose Koninklijke Ahold NV (AH) wants to hedge its interest rate risk by entering into a plain vanilla interest rate swap with Deutsche Bank AG (DB). AH proposes the following conditions regarding the swap:

- AH pays a fixed interest rate of 1.76% and receives 3 month Euribor (this makes the initial value of the swap zero)
- DB pays 3 month Euribor and receives a fixed rate of 1.76%
- Both fixed rate and floating rate payments occur quarterly
- The principal amount is 10,000,000 euro
- The maturity of the swap is 10 years
- All payments are made in euros

April 30, 2014 is both the valuation date and the start date of the swap, hence the first payments associated with the swap will be made on 30 July 2014. Note that no CSA underlying the swap is proposed, which means the swap would be uncollateralized. The bank and the corporate enter into negotiations to discuss the fair price of the swap. Both parties agree that the price of the swap involves discounting the floating leg payments and the fixed leg payments using the Eonia curve.
Besides, both parties recognize that since there is no CSA involved in the deal, both parties will have significant credit risk towards each other, hence a CVA and DVA will be incorporated in the fair price of the swap. Thereby, DB argues that funding cost may arise when it hedges the trade in the interbank market via a collateralized swap, as these cost are part of the construction of the deal, they should be incorporated in the price. Hence, the bank proposes the incorporation of a FVA for the expected funding costs the bank will have over the maturity of the swap. The bank would calculate the FVA along with BCVA, by modelling the CDS-bond basis, which should avoid double counting. Thereby, the bank will add a mark up to FVA for the internal transfer cost of the bank ($F = 0.005$). The case questions are, what should be the total value adjustment (TVA)? Where, $TVA = BCVA + FVA$, and how sensitive are BCVA, FVA and TVA to changes in wrong-way risk, credit spread levels and credit spread volatilities?

4.2 Data

We obtained all needed market data from Bloomberg, observed on April 30, 2014. To start with, we obtained the Eonia curve, which represents the market discount curve and the 3 month Euribor curve, which is used to calculate the floating leg of our swap. Thereby, we use historical Eonia spot rates and 3 month Euribor spot rates to model the Euribor-OIS spread. The historical rates are observed between April 30, 2010 and April 30, 2014. Furthermore, we obtained market euro at-the-money swaption volatilities to calibrate the G2++ model. Besides, to bootstrap market implied survival and default probabilities, we obtained the euro senior CDS curve for both Koninklijke Ahold NV and Deutsche Bank AG. Lastly, we also obtained current and historical (zero-coupon) bond yields of Deutsche Bank to calculate FVA. More detailed information regarding the market data can be found in Appendix A.

4.3 Research Approach

Firstly, we calibrate the model parameters of the G2++ model, the Euribor-OIS spread model, the CIR++ model and net funding cost model to our market data, according to the methodology described in respectively Subsections 3.1.3, 3.2.1, 3.3.2 and 3.5.3. Subsequently, we simultaneously simulate the relevant stochastic processes 1000 times. For each generated scenario, we calculate exposures, default probabilities and survival probabilities. By doing this, we can calculate the BCVA according to Eq. 3.35, where the expectations are approximated using Monte Carlo integration. Likewise, the FVA is calculated according to Eq. 3.39.
Chapter 5

Results

In this chapter we document the results of the case study by applying the described methodology. We provide calibration results including parameter estimations, relevant graphs and ratios to indicate the goodness of fit. Thereby, we provide the results of the impact analysis in the form of tables and graphs.

Without taking wrong- and right-way risk into account, we estimate a FVA of 40,661 euro and a BCVA of 26,126 euro adding up to a TVA of 66,787. Hence, by incorporating FVA and BCVA the fair value of the swap becomes $0 + 66,787 = 66,787$ euro. Thus, by incorporating counterparty credit risk and funding risk, Deutsche Bank can see the swap as an asset rather than a liability. On the contrary, Ahold can regard the swap as a liability. To explain this finding, we report the exposure profiles plotted in Figure 5.1.

![Expected Exposure (EE) and Negative Expected Exposure (NEE)](image)

Figure 5.1: Expected Exposure ($EE(0,T)$) and Negative Expected Exposure ($NEE(0,T)$) for Deutsche Bank corresponding with the receiver IRS described in the case study. Expected Positive Exposure ($EPE$) and Expected Negative Exposure ($ENE$) are the averages of respectively $EE$ and $NEE$. 
Clearly the NEE is on average higher in absolute value than the EE. This indicates that on average Ahold has more exposure to Deutsche Bank than vice versa. Thus, given that the two entities have similar survival probabilities (see Figure 5.6), the DVA term will be greater than the CVA term and so BCVA will be positive (26,126 euro). Likewise, the fact that ENE is higher than EPE in absolute value implies that it is more likely for Deutsche Bank to have a funding benefit than funding costs. Hence, there is an expected funding benefit, indicating a similar scenario as in Figure 3.2 will occur. This reasoning follows from the formula for FVA in Eq. 3.39. Therefore, we observe a positive FVA (40,661 euro). Recall that we defined: Fair Value = NPV + BCVA + FVA, so that both the positive BCVA term and the positive FVA will contribute to a higher Fair Value, i.e. the value of the swap taking into account both funding and credit risk.

5.1 Calibration

In this section the calibration results of our stochastic models are documented. We provide inter alia parameter estimations, relevant graphs and ratios to indicate the goodness of fit.

5.1.1 G2++ Model Parameters

In order to perform the calibration of the G2++ model parameters we use the methodology described in Subsection 3.1.3. We use the current market observed Euribor curve and euro at-the-money swaption volatilities to calibrate the G2++ model. Moreover, by varying the model parameters \((a, b, \sigma, \eta\) and \(\rho\)), we minimize the sum of squares of the percentage differences between the model and market implied swaption volatilities. The calibration produces the following parameter estimates:

\[
\begin{align*}
a &= 0.351447 & b &= 0.001812 & \sigma &= 0.010573 & \eta &= 0.008802 & \rho &= -0.988529. \\
\end{align*}
\]

To give an indication of the goodness of fit, we report in Figure 5.2 the calibration errors in the form of percentage differences between the model and market implied swaption volatilities.
The standard deviation of the calibration errors is 2.82%. Moreover, the absolute average mispricing regarding the calibration is 2.28%, which we consider fairly small.

### 5.1.2 Euribor-OIS Spread Model Parameters

We calibrate the Euribor-OIS spread model parameters according to the methodology described in Subsection 3.2.1. We use historical daily market data of Euribor and OIS bond prices over the past 4 years (1027 observations) to obtain the model parameters under $P$. Afterwards, we use current Euribor and OIS bond prices to determine the market price of risk $\lambda$. By doing this, we may adjust the real-world parameter estimates to risk-neutral parameter estimates. The resulting risk-neutral parameter estimates are

$$\kappa = 2.0771 \quad \mu = 0.0022 \quad \nu = 0.0464,$$

and for the market price of risk we obtain $\lambda = 1.3292$. To give an indication of the magnitude of the estimation error of $\lambda$ we also report the root mean squared error (RMSE), which equals 0.0081. Although, the RMSE is not negligibly small, we consider this estimation error to be acceptable. Moreover, we provide the model and market implied bond prices stemming from the calibration procedure in Figure 5.3.
5.1.3 CIR++ Model Parameters

To calibrate the CIR++ model, we first bootstrap market implied default intensities and survival probabilities from CDS data of Deutsche Bank AG and Koninklijke Ahold NV as described in Subsection 3.3.2. In Figures 5.4 and 5.5 we show the resulting market implied default intensities of respectively Deutsche Bank and Ahold.

![Plot showing market implied bond price $P_{MKT}$ and the (fitted) model implied bond price $P_{CIR}$ (evaluated at $\lambda = 1.3292$).]

![Plot showing market implied piecewise constant default intensity $\gamma$ of Deutsche Bank, stripped from CDS data on April 30, 2014.]

Figure 5.3: The market implied bond price $P_{MKT}$ and the (fitted) model implied bond price $P_{CIR}$ (evaluated at $\lambda = 1.3292$).
Figure 5.5: *Market implied piecewise constant default intensity $\gamma$ of Ahold, stripped from CDS data on April 30, 2014.*

From these graphs we may construct risk-neutral survival probabilities which we report in Figure 5.6. Note that the market regards Ahold slightly default riskier than Deutsche Bank over the long term (10 years). Next we use Eq. 3.24 to fit the term structure of credit spreads of both Deutsche Bank and Ahold to the CIR++ model.

Finally, we are left with the CIR parameter vector $\beta$ which can be calibrated to other securities. However, this is tricky since the current market for CDS options is lacking liquidity. Therefore, we choose reasonable values for $\beta$, in accordance with the estimates of [Brigo & Mercurio (2006)](http://example.com), which uses a hypothetical implied volatility surface of CDS options. We set

$$\kappa = 0.354201 \quad \mu = 0.001219 \quad \nu = 0.023819,$$

for both Ahold and Deutsche Bank. In Section 5.2 we vary $\mu$ and $\nu$ to analyse the impact of, respectively, credit spread levels and credit spread volatilities on BCVA and FVA. Calibrating the CIR++ model parameters to real market data is left for future research.

Figure 5.6: *Risk-neutral survival probabilities of Deutsche Bank (DB) and Ahold (AH)*
5.1.4 Net Funding Cost Model Parameters

In order to calibrate the net funding cost model parameters we use the methodology described in Section 3.5.3. We assume that $F$ equals 50 basis points and use historical market data of the Eonia spot rate, Deutsche Bank bond yields and Deutsche Bank CDS spreads to obtain model parameters under $\mathbb{P}$. Next, we use the current zero coupon bond, Eonia and CDS curve of Deutsche Bank to estimate the relevant market price of risk. The market price of risk is used to transform the model parameters under $\mathbb{P}$ to $\mathbb{Q}$. The resulting risk-neutral parameter estimates are

$$\kappa = 7.5796 \quad \mu = 0.0044 \quad \nu = 0.0940, \quad (5.4)$$

and the estimation of the market price of risk equals 3.6239. To give an indication of the goodness of fit of the estimation of $\lambda$ we also report the root mean squared error (RMSE), which equals 0.0047. Although, the RMSE is not negligibly small, we consider this estimation error to be acceptable. Besides, we provide the model and market implied bond prices stemming from the calibration procedure in Figure 5.7.

![Figure 5.7: The market implied bond price $P_{\text{MKT}}$ and the (fitted) model implied bond price $P_{\text{CIR}}$ (evaluated at $\lambda = 3.6239$).](image)

5.2 Impacts

Since the calculations of BCVA and FVA are highly model dependent, model risk is applicable. The dynamic features underlying the value adjustments impact the calculations and therefore it is interesting to examine these impacts. Moreover, we analyse the impact of wrong- and right-way risk, credit spread levels and credit spread volatilities on BCVA and FVA. The results are documented in...
5.2.1 Wrong- and Right-way Risk

Tables 5.1 to 5.4 and Figure 5.8 illustrate the impact of wrong- and right-way risk on BCVA and FVA. The value adjustments are calculated for a set of different wrong- and right-way risk scenarios. Moreover, we vary \( \bar{\rho}_j, j \in \{AH, DB\} \), where \( \bar{\rho} \) is defined as in Subsection 3.4.5.

We observe no clear pattern in the results. Besides, the variations in the value adjustments are fairly small. BCVA only alters between -22,707 and -28,341 euro, whereas FVA alters between -33,234 and -41,738 basis points. Hence, we notice a limited effect of wrong- and right-way risk on BCVA and FVA. Indeed, wrong- and right-way risk will have the impact of increasing EE and decreasing NEE or vice versa. This balances both BCVA and FVA. The results confirm these principles.

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<th>( \bar{\rho}_{AH} )</th>
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<th>FVA</th>
<th>TVA</th>
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5.2.2 Credit Spread Levels

Credit spread levels reflect the creditworthiness of entities and are obvious risk factors in over-the-counter derivatives transactions. We analyse the impact of credit spread levels on BCVA and FVA. The results are documented in Tables 5.5 to 5.7.

We vary $\mu_j$, $j \in \{AH, DB\}$, i.e. the long term mean parameter in the CIR++ default intensity model, which corresponds to shocking the long term mean of the credit spreads of both DB and AH. Moreover, we examine three different scenarios: varying $\mu_{DB}$, varying $\mu_{AH}$ and varying both $\mu_{DB}$ and $\mu_{AH}$. The former case is illustrated in Table 5.5. We notice a strong effect on BCVA, which increases as $\mu_{DB}$ increases. This is expected, since DB’s own credit spread rises and the counterparty’s credit spread stays the same. Ceteris paribus, this implies that DVA will increase and CVA will stay the same, thence BCVA(=DVA-CVA) will increase. In Table 5.6 we keep $\mu_{DB}$ fixed and vary $\mu_{AH}$. In this case BCVA increases as $\mu_{AH}$ increases. Indeed, CVA will increase due to the worsened creditworthiness of AH.

FVA slightly decreases when the credit spread level of one of the two entities increases. Understandably, when the survival probability of one of the two entities decreases the deal is expected to terminate earlier, hence expected funding cost are smaller. This effect is visualized in Figure 5.9 and highlighted in Table 5.7 where we vary both $\mu_{DB}$ and $\mu_{AH}$. Notice that FVA strongly decreases in this case.
Table 5.5: Varying $\mu_{DB}$

<table>
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<th>$\mu_{DB}$</th>
<th>$\mu_{AH}$</th>
<th>BCVA</th>
<th>FVA</th>
<th>TVA</th>
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Table 5.6: Varying $\mu_{AH}$

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Table 5.7: Varying $\mu_{DB} = \mu_{AH}$

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Figure 5.9: Plot of FVA according to the results in Table 5.5 and 5.7, the x-axis denotes the increase in $\mu$.

5.2.3 Credit Spread Volatilities

The impact of credit spread volatility on BCVA and FVA is often neglected in the literature by assuming constant credit spreads. Tables 5.8 to 5.10 report (a part of) the panel of results regarding the impact of credit volatility on BCVA and FVA.

We study three different credit spread volatility scenarios by changing $\nu_j$, $j \in \{AH, DB\}$, i.e. the main volatility parameter in the CIR++ default intensity model. The results confirm that the impact of credit spread volatility on BCVA and FVA is non negligible. The impact on BCVA is significant and similar to the previous case, where we varied $\mu_j$. In Table 5.10 FVA can be seen to decrease as
the credit spread volatility of both entities increases. Interestingly, BCVA has the opposite effect. This results in a fairly stable TVA. Hence, including FVA in the valuation of an IRS can be seen as a natural hedge against credit spread volatility. This finding is illustrated in Figure 5.10, where we plotted the estimated value adjustments in the case we vary $\nu_{DB} = \nu_{AH}$.

Table 5.8: Varying $\nu_{DB}$

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Table 5.9: Varying $\nu_{AH}$

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<th>$\nu_{DB}$</th>
<th>$\nu_{AH}$</th>
<th>BCVA</th>
<th>FVA</th>
<th>TVA</th>
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Table 5.10: Varying $\nu_{DB} = \nu_{AH}$

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<thead>
<tr>
<th>$\nu_{DB}$</th>
<th>$\nu_{AH}$</th>
<th>BCVA</th>
<th>FVA</th>
<th>TVA</th>
</tr>
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</table>

Figure 5.10: Plot of the estimated value adjustments in the case we vary $\nu_{DB} = \nu_{AH}$, the x-axis denotes the increase in both $\nu_{DB}$ and $\nu_{AH}$.
Chapter 6

Conclusion

The main result of this thesis is an innovative framework for modelling funding value adjustment. We motivated and examined this framework in the form of a case study considering a plain vanilla interest rate swap. Nevertheless, the proposed framework for modelling funding value adjustment can be applied to many different derivative products.

In general our FVA estimates are significant and quite sensitive to parameter dynamics. Moreover, the impact of wrong- and right-way risk is found to be limited, whereas credit spread levels have a structured effect on both BCVA and FVA. Besides, the often neglected impact of credit spread volatility is found to be non negligible. Therefore there is a need to rigorously model these dynamical features. We have chosen a G2++ model for interest rates, a CIR model for the Euribor-OIS spread, a CIR++ model for credit spreads and a CIR model for net funding cost. We document our findings in the form of tables and graphs.

Still, there are great opportunities to extent the modelling framework in future research. The academic field of FVA is far from discovered, hence plenty of extensions are possible. To start with, one can model the dependence structure between funding spreads and credit spreads (i.e. wrong-way funding risk). For example, a correlation between the treasury’s internal lending rate and the bank’s credit spread can be considered: does the treasury’s internal lending rate rises as the creditworthiness of the bank weakens? Besides, one can extend the modelling framework in order to calculate FVA for netting sets or investigate the effect of different funding policies on FVA. Likewise, the impact on FVA of different credit spread and interest rate models can be examined. Another extension would be to implement discretization schemes that diminish districtization errors and give more accurate estimations of BCVA and FVA. Lastly, one can introduce default correlation between entities by making use of copulas.

It may be clear that the implementation of FVA is complex and that there is a great need for a market practice regarding FVA. Indeed, the price of a derivative including FVA is different for each entity since each entity has its own funding policy. Therefore, there is no law of one price regarding derivatives pricing including funding risk and deals will be made through negotiations. Thus, FVA cannot be estimated correctly based on market observable quantities.

International Financial Reporting Standards 13 already forces banks and other derivative dealers to
incorporate CVA and DVA. However, it may be clear that similar obligatory accounting practices are inapplicable to FVA in the current market environment. The Basel Committee on Banking Supervision can consider to force (big) bank to make their internal lending rates public. This could be a first step towards a market observable FVA and may foster the commencement of a market practice regarding FVA. Likewise, the origination of an industry average of the relevant treasury’s internal lending rate of big banks could also be a possibility.

This thesis confirms the tremendous impact of the global credit crisis on the key concepts underlying the valuation of derivatives contracts. As a result, the valuation of derivatives contracts is much debated today making even the valuation of the simplest derivatives contracts a challenging task. This shows how significant the effect of post-crisis realities such as the Libor-OIS spread can be.
Appendix A

Market Data

<table>
<thead>
<tr>
<th>Mth/Pay</th>
<th>Market Rate</th>
<th>Spot Rate</th>
<th>Discount</th>
<th>Source</th>
<th>DCC</th>
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Figure A.1: Market data of the 3 month Euribor and Eonia rate in respectively the right and left figure (obtained from Bloomberg on April 30, 2014).
Figure A.2: Market at-the-money swaption volatilities, obtained from Bloomberg on April 30, 2014. Each column indicates the tenor of the swaption, whereas each row denotes the maturity of the swaption.

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<th>3Yr</th>
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<th>6Yr</th>
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<tr>
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Figure A.3: Market CDS curves of Deutsche Bank AG and Koninklijke Ahold NV in respectively the left and right figure (CDS spreads are in basispoints). The data is obtained from Bloomberg on April 30, 2014.

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<tr>
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<th>Koninklijke Ahold NV EUR Sr CDS Curve</th>
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<td>Tenor Mid Spread</td>
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<td>6M 14.1</td>
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Figure A.4: Bond yield curve of Deutsche Bank AG, built with senior unsecured fixed rate bonds issued by Deutsche Bank AG (obtained from Bloomberg on April 30, 2014).

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<td>Yield percentage</td>
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References


